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ON SOME PHENOMENA OBSERVED IN BISMUTH PLATES WHEN PLACED IN A STEADY MAGNETIC FIELD.

I.

EXPERIMENTS ON THE TRANSVERSE EFFECT

AND ON

SOME RELATED ACTIONS IN BISMUTH.

II.

RELATION BETWEEN THE VARIATION OF RESISTANCE IN BISMUTH IN A STEADY MAGNETIC FIELD AND THE ROTATORY OR TRANSVERSE EFFECT.

WITH TWO PLATES.

THESIS PRESENTED BY JOHN C. BEATTIE

FOR

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VI.—*Experiments on the Transverse Effect and on some Related Actions in Bismuth.* By J. C. BEATTIE. (With a Plate.)

(Read 17th December 1894.)

SECTION I.—INTRODUCTION.

CLERK MAXWELL, in his *Electricity and Magnetism*, vol. i. § 304, makes the following remark about the rotatory coefficient :—" It should be found, if anywhere, in magnets which have a polarisation in one direction, probably due to a rotational phenomenon in the substance."

The current which should arise from such a coefficient was first observed by HALL. He passed a current through a strip of metal; he then found two points on opposite sides of the strip, which, while the current was flowing, were at the same potential, and which therefore indicated no current when joined to a galvanometer. The plate was next brought into a uniform magnetic field, and when everything was steady the two points previously at the same potential were no longer so, and a current flowed through the galvanometer. This effect is observable in all conductors.

KUNDT * has shown that in iron, nickel, cobalt, it is proportional to the magnetisation. Whether this is true for the diamagnetic metals has not, so far as I know, been definitely settled yet. But, should this be proved, we have a comparatively easy method for studying the magnetisation in these metals.

Another phenomenon which may advantageously be studied by means of the transverse effect is the variation of resistance of conductors carrying a current in a magnetic field. GOLDHAMMER † has shown in another way that the increase or decrease of the resistance in bismuth is proportional to the square of the magnetisation, and suggests that the same may be true for cobalt and nickel. Evidently the proportionality or non-proportionality for these two latter metals can be settled at once by comparing the variation of resistance and the transverse effect at the same field strength; and, similarly, the same method can be employed to show whether or not the variation of resistance bears any relation to the magnetisation in all cases where it has first been proved that the magnetisation and the transverse effect are proportional. So far as I know, this method has not as yet been tried experimentally. I propose in another paper to give some results relating to this matter.

In bismuth the transverse effect has not yet been proved to be proportional to the magnetisation; nor, indeed, is it certain that the so-called transverse effect in bismuth is a pure Hall effect, ‡ or is caused by a number of separate effects. As I shall show later, the transverse effect in most cases is really the sum of three effects.

* WIEDEMANN'S *Annalen*, 1893, Bd. 49, S. 257.

† WIEDEMANN'S *Annalen*, 1889, Bd. 36.

‡ By Hall effect is meant a transverse effect proportional to the first power of the magnetisation. See " On Relation between the Variation of Resistance in Bismuth, &c.," *Trans. R.S.E.*, vol. xxxviii.

The following experiments were carried on in the Physicalisches Institut, Muenchen; and I have to thank Professor BOLTZMANN for the trouble he put himself to, for his suggestions, and for placing the whole resources of his laboratory at my disposal.

The plates used were cast from two separate quantities of ordinary mercantile bismuth. In some instances they were cooled quickly, in others slowly. The thicknesses varied from three to one millimetre; the ratio of length to breadth was about three to one as the plates were originally used; afterwards these dimensions were considerably modified.

The galvanometer used was a Wiedemann, with a Siemen's well-formed magnet.

The electro-magnet used for the creation of the magnetic field consisted of two cylinders of soft iron 60 cm. long, 16 cm. in diameter, placed on a parallelepiped of the same material 63 cm. long, 20 cm. high, 20 cm. broad. The shoes were formed by two blocks 16 cm. square, 20 cm. long, to which truncated cones were fixed with a base diameter of 16 cm., a summit diameter of 6 cm. Each cylinder was surrounded by two spools, round which the copper wires were wound. Diameter of the wire 2·5 mm.; the total length of wire was 3850 mm.; the number of windings 5951. (Cp. fig. A, Plate P.)

The current to the electro-magnet was supplied by an accumulator battery of 56 cells.

The strength of the field was measured by Verdet's method. A wire was arranged in the form of a square, the ends were inserted into the galvanometer circuit, and when the electro-magnet was on, the square which was kept perpendicular to the lines of force was pulled quickly out of the field.

The readings thus obtained were compared with those obtained from an earth inductor inserted in the same circuit, and the strength of the field in absolute units arrived at in the usual way. To get the strength of the field in absolute units, the numbers given as field strengths in the results must be multiplied by 138·5.

The strength of the current which flows in the direction of the plate's length—and which will be called the primary current—was measured at the beginning and end of each series of experiments. For this purpose a thick copper wire was inserted in the primary circuit. To two points of this, copper wires were soldered, which, by means of a commutator could be placed in the galvanometer circuit when necessary.

The electro-magnet was so placed that it exercised a minimum effect on the galvanometer, which was at a distance of thirty or forty feet. The magnet and primary currents could both be reversed by commutators; the number of readings necessary to eliminate disturbing effects was thus four. The average of the four readings was divided by the primary current strength: this quantity is called later the transverse effect.

The positive direction of the transverse effect is so defined: Let the plate of bismuth be supposed to be in the plane of the paper with the north pole of the magnet above, the south below, the paper. Then, if in going from the point where the primary current enters to that where the transverse current enters the motion is counter clock, we call the transverse effect positive.

In diagram B, Plate P, with dotted circle to represent north pole above the paper, the transverse effect is positive.

To both ends of the plates strips of copper of the same breadth and thickness were soldered; to these latter, wires were soldered, which lead to the accumulators giving the primary current.

To two points in the middle of the sides of the bismuth plates wires were soldered; each wire was doubled on itself, the point of contact with the bismuth forming the bottom of a V. Between the arms of the V mica was inserted to insure insulation. Both arms were kept in the plane of the bismuth plate and perpendicular to its length. One arm of each was joined to the galvanometer; the other led to a mercury pool in the first series of experiments, in the later ones it was unconnected. (Fig. C, Plate P.)

SECTION II.—ON THE EFFECT ON THE TRANSVERSE CURRENT OF INSERTING A SHUNT WHOSE RESISTANCE IS OF THE SAME ORDER OF MAGNITUDE AS THAT OF THE PLATE.

The transverse effect has up till now always been measured with a galvanometer whose resistance was many times greater than that of the plate of metal experimented upon. The question arises, How will this current be affected when a shunt is inserted between the transverse electrodes whose resistance is of the same order of magnitude as that of the plate? If the plate when placed in a steady magnetic field behaves as a cell with constant electromotive force would do, it will divide according to Ohm's law; if, on the other hand, it behaves as a cell whose electromotive force is not constant, the current will not be obtainable from the equation—

$$\text{Current} = \frac{\text{Electromotive Force}}{\text{Resistance}}.$$

For example, Prof. LOMMEL, in his paper "Sichtbare Darstellung der äquipotentialen Linien in durchströmten Platten. Erklärung des Hall'schen Phänomens,"* has proposed a formula, according to which the insertion of a shunt between the two transverse electrodes would not affect the reading on a galvanometer whose resistance is great—compared with the sum of the resistances of the bismuth plate and of this shunt.

Each plate experimented upon was placed between the poles of the electro-magnet, perpendicular to the lines of magnetic induction.

Before the magnet was put on, a current from the accumulators at A was sent through the plate. Two points, E and D, as near the middle as possible, were then found, so that when wires joined them to the galvanometer G, no current passed. From E and D two other wires lead to the mercury pools L and M respectively. Should it be found impossible to find two points at the same potential, the current which goes through the galvanometer circuit can be eliminated by joining E and N or D and N, as the case may be, and inserting a suitable resistance. (Cp. fig. D, Plate P.)

* *Sitzungsbericht der Köng. bayerischen Akedamie der Wissenchaft*, 1892, Bd. xxii. Heft iii. § 371.

A series of five experiments was made with each field strength. In 1st, 3rd, and 5th E L M D was open, in 2nd and 4th E L M D was closed.* The average of the first three was then divided by that of the 2nd and 4th.

Next, the resistance of the bismuth plate was measured when the electro-magnet was on. A current was sent by A in the direction A L E D M A, or *vice versâ*, and E and D were joined to the galvanometer; four readings were taken—the resistances of the copper wires L E, M D and of the short wire L M—the total being of the same order of magnitude as that of the bismuth plate. These measurements were made at the beginning and end of each series of five experiments.

Let C be the transverse current when E L M D is open, let S be the resistance of the shunt E L M D, n that of the bismuth plate. Then theoretically we have

$$\frac{\text{Current when E L M D is open}}{\text{Current when E L M D is closed}} = \frac{\mathrm{C}}{\dfrac{c/n}{1/n+/1s}} = 1+\tfrac{n}{s}$$

But since we have measured n and s directly, we can calculate $1+\frac{n}{s}$; the calculated and the observed values will agree, if the transverse effect is of the same nature as the current obtained from a cell of constant electromotive force.

The following are some of the results obtained:—

PLATE (IA).

Length, ...	5·63 cm.
Breadth, ...	2·9525 „
Thickness, ...	0·19413 „

This plate was quickly cooled in casting; the temperature of the room was in all the experiments about 15° C. Made from first supply of bismuth.

Field Strength.	Trans. Current.	Shunted Trans. Current.	Trans. Current. / Shunted Trans.	$1+\frac{n}{s}$ Calculated.
5·4	−0·04412	−0·030880	1·428	1·431
24·0	−0·15364	−0·10518	1·460	1·458
52·2	−0·20186	−0·13526	1·492	1·492
98·0	−0·21016	−0·1328	1·582	1·558
114·2	−0·19179	−0·12131	1·581	1·581
131·0	−0·17829	−0·11419	1·562	1·60

PLATE (IB).

Length, ...	6·045 cm.
Breadth, ...	2·58 „
Thickness, ...	0·12235 „

Slowly cooled. Made from first supply of bismuth.

* The galvanometer reading obtained in this case, divided by the strength of the primary current, is called in the results the shunted transverse.

Feld Strength.	Trans. Current.	Shunted Trans. Current.	$\frac{\text{Trans. Current.}}{\text{Shunted Trans.}}$	$1+\frac{n}{s}$ Calculated.
118·0	−0·14253	−0·0666	2·138	2·118
140·0	−0·20239	−0·09329	2·169	2·150
148·0	−0·22356	−0·10174	2·197	2·159

PLATE II.

Length,	. . . . .	6·26 cm.
Breadth,	. . . . .	2·945 „
Thickness,	. . . . .	0·149725 „

Slowly cooled. Made from first supply of bismuth.

Field Strength.	Trans. Effect.	Trans. Effect with Shunt.	$\frac{\text{Trans. Effect.}}{\text{Trans. Shunted.}}$	$1+\frac{n}{s}$ Calculated.
36·0	−0·19674	−0·09929	1·981	1·947
61·0	−0·25164	−0·14596	1·724	1·76
120·0	−0·30315	−0·13744	2·203	2·186
137·8	−0·30779	−0·12963	2·374	2·357
146·0	−0·31051	−0·11821	2·626	2·489

The dimensions of the plate were next altered; in particular the thickness was considerably reduced by planing. It was now

0·0827 cm.

Field Strength.	Trans. Effect.	Trans. Effect with Shunt.	$\frac{\text{Trans. Effect.}}{\text{Trans. Shunted.}}$	$1+\frac{n}{s}$ Calculated.
28·2	−0·37519	−0·18141	2·068	2·062
66·7	−0·57451	−0·25566	2·247	2·25
147·0	−0·70719	−0·25157	2·8111	2·826

PLATE III.

This plate was of pure bismuth, specially prepared by Professor Classen in Aachen.

Length,	. . . . .	1·99 cm.
Breadth,	. . . . .	1·075 „
Thickness,	. . . . .	0·126975 „

Field Strength.	Trans. Effect.	Trans. Effect with Shunt.	$\frac{\text{Trans. Effect.}}{\text{Trans. Shunted.}}$	$1+\frac{n}{s}$ Calculated.
9·4	−0·05219	−0·03953	1·320	1·327
55·0	−0·16162	−0·113	1·430	1·415
100·0	−0·22199	−0·14378	1·543	1·539
138·2	−0·27166	−0·16375	1·658	1·660
146·0	−0·2812	−0·16676	1·6777	1·679
153·0	−0·28849	−0·17034	1·6936	1·6929

PLATE VI.

Rapidly cooled; made from first supply of bismuth.

Length,	5·81 cm.
Breadth,	2·9935 „
Thickness,	0·14105 „

Field.	Trans. Current.	Trans. Current Shunted.	$\dfrac{\text{Trans. Current.}}{\text{Trans. Current Shunted.}}$	$1+\frac{n}{s}$ Calculated.
48·4	−0·0185	−0·0075	2·4666	2·4671
87·2	+0·05646	+0·02206	2·5628	2·5621
118·0	+0·11916	+0·0461	2·5845	2·558

PLATE VII.

Slowly cooled; made from first supply of bismuth.

Length,	6·032
Breadth,	1·507
Thickness,	0·06713

Field.	Trans. Current.	Trans. Current Shunted.	$\dfrac{\text{Trans. Current.}}{\text{Trans. Current Shunted.}}$	$1+\frac{n}{s}$ Calculated.
17·8	−0·2536	−0·00883	2·872	2·8979
121·6	+0·1852	+0·06045	3·0637	3·0667
140·0	+0·22024	+0·07243	3·0407	3·0805
147·8	+0·2484	+0·08184	3·0352	3·0543

PLATE VIII.

Quickly cooled; made from new supply of bismuth.

Length,	5·99 cm.
Breadth,	3·03 „
Thickness,	0·3153 „

Field.	Trans. Current.	Trans. Current Shunted.	$\dfrac{\text{Trans. Current.}}{\text{Trans. Current Shunted.}}$	$1+\frac{n}{s}$ Calculated.
41·4	−0·09817	−0·0651	1·5077	1·5016
80·2	−0·108	−0·06846	1·5775	1·5799
100·0	−0·09986	−0·06144	1·6253	1·6251
123·3	−0·09026	−0·05554	1·6251	1·6475

It will be seen from a comparison of the last two columns in the different results that the agreement between the observed and the calculated values of $1+\frac{n}{s}$ is in most cases close; the discrepancies can be quite well accounted for by experimental errors in measuring such small resistances.

Section III.—On the Change of Sign of the Transverse Effect.

In Plate I. it was noticed that the transverse effect attained a maximum and then decreased steadily with increasing fields. Other plates were then made, to see if this result was observable in them; and in some of them the maximum was reached with comparatively weak fields. With stronger fields the transverse current decreased, till finally it vanished; with still stronger fields it reappeared again, but with the opposite direction.

With Plate VI., as originally prepared, the following results were obtained :—

Field.	Transverse Effect.	Sign.	Rotatory Coefficient.
3·5	·01179	−	3·83
5·0	·01648	−	3·74
11·5	·0302	−	2·99
19·0	·03595	−	2·15
22·0	·03993	−	2·06
32·0	·03787	−	1·35
40·0	·03183	−	0·88
48·4	·0185	−	0·43
53·0	·01625	−	0·35
60·0	·0032	−	0·06
65·0	·0057	+	0·1
77·6	·03432	+	0·5
87·2	·05646	+	0·74
118·0	·11916	+	1·15
127·2	·14305	+	1·28

Here the transverse effect is at first negative; it increases, till with a field strength between $22 \times 138{\cdot}5$ and $32 \times 138{\cdot}5$ in c.g.s. units, it reaches a maximum. After that it decreases and finally vanishes between 60 and 65. It begins again, however, with increasing fields, and continues to increase; but now it has the opposite sign. The rotatory coefficient has its greatest—and negative—value with the weakest field.

This same plate was next shortened and narrowed, but the thickness kept as before. The transverse effect was again observed; it was still the same in character. It vanished with the same field strength as in the previous experiment; and with weak fields was negative, with strong positive.

Next the plate was made thinner by planing, and the following results were obtained :—

Plate VI.

Length,	4·22	cm.
Breadth,	2·36	"
Thickness,	0·0657	"

Field.	Transverse Effect.	Sign.	Rotatory Coefficient.
23·4	0·08714	−	1·97
29·0	0·09326	−	1·70
35·2	0·08958	−	1·35
41·8	0·08033	−	1·02
45·0	0·06857	−	0·081
69·0	Not observed.		
80·0	0·03144	+	0·21
102·8	0·12287	+	0·63
110·0	0·20363	+	0·88

A comparison of these results with those obtained with the original plate shows that the maximum negative effect is reached with a higher field, and that the field strength for which the effect vanishes is also higher. If we take the magnetic force as abscissa, the transverse effect as ordinate, we may express the result by stating that the curve giving the relation between the two has been moved, so that it cuts the axis at a point farther along in the positive direction.

See graph of curve giving relation between transverse effect and field strength in fig. 3, where A is the curve for the plate as originally cast, B that after it was hammered, C that after it was planed down.

Finally, the dimensions of the plate were again slightly modified, and, in addition, it was hammered.

PLATE VI.

Length,	3·25	cm.
Breadth,	1·24	„
Thickness,	0·0657	„

Field.	Transverse Effect.	Sign.	Rotatory Coefficient.
49·0	0·1075	−	1·16
66·1	0·08014	−	0·64
76·0	0·0394	−	0·27
85·0	Not observed.		
123·1	0·12246	+	0·527
134·0	0·1657	+	0·66

In this instance the reversal of sign takes place with a still stronger field. An attempt to further thin the plate proved abortive; it was now so brittle that planing caused it to break.

In Plate VII. the reversal was also observed in the plate as originally made; the effect disappeared with a field strength of $43 \times 138{\cdot}5$ c.g.s. It was then halved and the transverse effect for both halves was observed, and was found to vanish for the same field strength. Finally, one half was hammered; the same results—negative for the weaker fields, positive for the higher—were obtained, but the vanishing did not now take place until a field strength $60 \times 138{\cdot}5$ was reached.

Plate IB also showed this reversal. For the original plate the following results were obtained :—

PLATE IB.

Length,	6·045	cm.
Breadth,	2·58	„
Thickness,	0·12235	„

Field.	Transverse Effect.	Sign.	Rotatory Coefficient.
8·0	0·0302	−	3·72
15·1	0·05276	−	3·45
28·0	0·05517	−	1·94
38·0	0·0484	−	1·26
59·0	0·0218	−	0·36
63·0	0·003	−	0·046
75·6	0·02229	−	0·29
118·0	0·14253	+	1·19
140·0	0·20239	+	1·43
148·0	0·22356	+	1·49

The transverse effect is first negative ; it increases and reaches its maximum negative effect with field strength 28 (about). Afterwards it decreases and vanishes with field 63 (about). It again appears and increases for all other fields higher than 63, but now has the opposite direction.

This plate was next varied in length and in breadth, but the same thickness was retained ; and the transverse effect was found to vanish for the same field strength.

The plate was then hammered, and it was found that the transverse effect did not vanish until a field strength of about 80 was reached ; the field strength by which the maximum negative effect was reached was also greater.

The plate was now made thinner by planing—

Length,	4·47	cm.
Breadth,	2·08	„
Thickness,	0·0665	„

Field Strength.	Transverse Effect.	Sign.	Rotatory Coefficient.
26·0	0·21699	−	4·48
38·0	0·22231	−	3·14
49·0	0·21172	−	2·32
62·0	0·13521	−	1·17
87·0	0·4749	−	0·29
97·0	Not Observed.		
101·0	0·0376	+	0·199
117·8	0·17013	+	0·77
135·0	0·27122	+	1·08
145·0	0·33084	+	1·22

We see that a still stronger field is now required to make the transverse effect vanish; and for the maximum negative effect also a stronger field is necessary than in the former cases.

Finally, the plate was again hammered, and the following results obtained :—

Field Strength.	Transverse Effect.	Sign.	Rotatory Coefficient.
50·0	0·23289	−	2·5
70·0	0·16731	−	1·28
100·0	0·01837	−	0·098
105·6	0·02256	+	0·115
123·5	0·11556	+	0·502
143·0	0·21119	+	0·79

which again shows a considerable increase in the field necessary to reverse the direction of the transverse current.

The reversal of direction was not observed in Plates II. and III., nor was a maximum effect reached in these two plates: In Plate I., again, no reversal was found, but a maximum effect was reached with a field strength a little over 100.

Another series of plates was now made from a new supply of mercantile bismuth. Two Plates, VIII. and IX., were made each about 3 mm. thick; VIII. was cooled quickly, IX. slowly. The transverse effect was negative throughout; it reached a maximum in both cases, and then began to decrease; but it could not be made to vanish by field strengths at disposal.

Two other Plates, X. and XI., were made, each about 1·5 mm. thick; X. was cooled quickly, XI. slowly. In these two plates the transverse current vanished, and with higher fields had the opposite sign positive.

Another Plate, XII., was made in the form of a cross; to the two arms of the cross the galvanometer wires were soldered, and the effect of the soldering on the plate as a whole minimised. (Cp. fig. E.) With this plate the following results were obtained :—

PLATE XII.

Length, ...	6·22
Breadth, ...	1·85
Thickness, ...	0·10462

Field Strength.	Transverse Effect.	Sign.	Rotatory Coefficient.
25·2	0·17158	−	5·74
34·6	0·18389	−	4·48
45·5	0·18115	−	3·36
88·3	0·05617	−	0·538
103·0	0·0119	+	0·097
116·5	0·06051	+	0·438
137·0	0·16766	+	1·033
145·0	0·20602	+	1·199

We may sum up the results as follows:—With thick plates the transverse current does not change its direction for any strength of field, though in some cases a maximum value is reached and passed; nor can the change of direction be brought about by planing, hammering, or modifying the dimensions of the plates. Cp. fig. 1, which gives the relation between field and effect for Plate IA, and fig. 2, which gives the same for Plate II.

With thinner plates the transverse current is positive for strong fields, negative for weak ones. The field strength at which vanishing takes place is the same for the same plate, so long as it is modified only in length and breadth; but if the plate be planed down, the field at which the current vanishes is stronger than in the original case. Similarly, if the original plate be hammered, the field required to produce vanishing is stronger: a combination of hammering and planing raises very considerably the strength of the field required. From a comparison of the results, it will be seen that in different plates the transverse current vanishes for different fields. Cp. fig. 3, where the three curves give the relation between field strength and transverse effect for Plate IB; A refers to the original plate, B to the same plate hammered, C to it after planing.

This reversal of the transverse current has already been observed by VON ETTINGSHAUSEN and NERNST* in an amalgam of bismuth and tin.

In the one certainly pure bismuth, Plate III., the ratio of the increase of resistance to the square of the transverse effect, was practically constant; this is as it should be, if in diamagnetic bodies the transverse effect is, as in the magnetic metals, proportional to the magnetisation I, and the increase of resistance proportional to, I^2. If we start from this and apply it to those plates in which the transverse effect vanishes, we find that our facts do not tally with our assumptions. For if the transverse current be proportional to the magnetisation, when the former vanishes, so must the latter and so must the increase of resistance too: that is, when the transverse current vanishes, the resistance of the bismuth at the same field strength must be the same as when no field is present. But for Plate IB. the following results were obtained:—

Field Strength.	Resistance Proportional to.	Transverse Effect.
0·0	254·0	0
28·0	265·4	− 0·5517
63·0	280·1	− 0·003
118·0	300·2	+ 0·14253
147·7	312·0	+ 0·22356

That is, the transverse effect vanishes at about 63, but the resistance increase is at the same field strength quite perceptible. Another effect observed in all the plates and which will be later described, supports the view that the increase of resistance does not vanish with

* *Sitz. bericht der kais. Akad. der Wissenschaft*, ii. Abth., 1887, Bd. 96.

the transverse effect. From this we may draw three conclusions:—(1) The two assumptions are both wrong; (2) one is wrong; (3) or we may still suppose both true, and assume that in bismuth two constants with opposite signs are involved in the transverse effect. That is, instead of assuming that it is proportional to the vector product of the primary current and the magnetisation, we assume that it is the vector product of the primary current and $(c_1 I + c_2 I^3)$.

In the first case we may write the electromotive force

$$e = c_1 Vu I$$

where c_1 is negative for bismuth and those metals which have a negative transverse effect; positive for those which have a positive effect.

In the second case

$$e = Vu(c_1 I + c_2 I^3)$$

where c_1 is the same constant as before, c_2 is another constant positive in the first class of substances negative in the second. In those substances in which the transverse effect is proportional to the magnetisation, c_2 is infinitesimally small in comparison with c_1; in bismuth and any other substances where this is not the case, c_2 has such a value that for sufficiently high fields the transverse effect may vanish, and for still higher reverse its direction. Similarly, c_2 might be of such magnitude that the transverse effect did not vanish, but still reached a maximum value, and then began to decrease as in Plate IA, fig. 1.

The validity of this assumption could be tested by determining the magnetisation directly, and thus determining c_1 and c_2 for different field strengths.

Section IV.—On Effects other than the Transverse Effect proper.

Two other such effects were observed. The first was evident in the whole of the plates experimented upon. In the plate of pure bismuth, III., it was such, that when the apparatus was arranged, as in diagram (D), in passing from the entrance of the primary current at B to that of the effect at D, the motion was counter-clockwise. It changed with the change in direction of the primary current, but not with the reversal of the magnet. Thus, with one arrangement of the magnet, it acted against the transverse current; in the other with it. In Plate III. it acted against the transverse current when the north pole of the magnet was in front of the diagram, with it when the south pole was in front.

The following results were obtained with Plate III.:—

Field, . .	55·0	100·0	138·2	153·0
Effect, . .	0·03613	0·08908	0·14475	0·16332

To find how this effect varied with the primary current strength, the field was kept constant, while the primary was varied :—

Primary Current Proportional to.	Effect in Scale Parts.	Effect. / Primary.
60·0	90·0	1·5
124·1	177·6	1·43
187·0	269·4	1·44
373·0	515·4	1·39

or the effect is, for the currents used, practically directly proportional to the primary current.

This effect had the same direction in Plate II., for which the following results were obtained :—

Field, . . .	36·0	61·0	120·0	137·0	146·0
Effect, . . .	0	0·0045	0·0509	0·07175	0·084

In Plate XII. again the direction of the effect was the opposite. In the other plates it had sometimes the one direction, sometimes the other ; indeed, after a plate had been planed or hammered, it sometimes had the opposite direction to that in the original plate. So long as it was less in magnitude than the transverse effect proper, its disturbing influence was eliminated by making four different experiments, according as the direction of the field or the primary current was varied. Should it, however, have a greater value than the transverse current, this was no longer the case ; and when the transverse current vanished, it alone was observable ; it increased in all cases with the field strength, and in no case did it change sign, unless the plate was modified.

The existence of an effect whose direction can in no case be predicted follows from the general equations for a non-isotropic body. For, suppose we have an isotropic body which is brought into a magnetic field and carries a current, it is acted on by mechanical forces. The body becomes anisotropic, and the transverse coefficients of resistance are brought into play ; a transverse current flows, whose direction is determined by the structure of the body. Or, suppose the body to be originally non-isotropic, a transverse current will be observed with no magnetic field present ; this, however, can be eliminated by the insertion of a proper resistance. When the magnet is excited, the transverse resistance is modified, so that the inserted resistance no longer balances it. Result is, the transverse current again appears. The fact that it does not depend on the direction of the field shows that the resistance concerned is proportional to some even function of the magnetisation.

The equations for such a body would be

$$X = r_{11}u + r_{12}v + r_{13}w$$
$$Y = r_{12}u + r_{22}v + r_{23}w$$
$$Z = r_{13}u + r_{23}v + r_{33}w$$

If to this we add the fact that a magnetic field gives rise to a rotatory coefficient as well, which is an odd function of the magnetisation, we have the most general equations

$$X = r_{11}u + r_{12}v + r_{13}w + T_3v - T_2w$$
$$Y = r_{12}u + r_{22}v + r_{23}w + T_1w - T_3u$$
$$Z = r_{13}u + r_{23}v + r_{33}w + T_2u - T_1v$$

where x, y, z are the components of the electromotive force parallel the three axes; u, v, w the components of the current; p_{11}, r_{22}, r_{33} the direct resistances; r_{12}, r_{13}, r_{23} the transverse resistances; T_1, T_2, T_3 the rotatory resistances.

The second effect was not observed in all the plates. Its presence was observed by the gradual decline of the galvanometer deflexion of the transverse current, which lasted for about a minute, when a steady state was usually reached. It was measured in the following manner:—After the steady transverse reading had been taken, the electromagnet was kept on, the primary current was broken, and the galvanometer immediately inserted. The reading thus obtained was usually small and died away gradually. In every instance it was oppositely directed to the transverse current.

PLATE I.

Length,	5·45	cm.
Breadth,	2·96	„
Thickness,	0·1305	„

Field,	9·6	36·8	58·0	119·0	131·5	141
Effect,	0·0069	0·0319	0·0437	0·0429	0·0381	0·0371

The numbers given under "effect" are here, as before the galvanometer reading, divided by the primary current.

The same plate was previously used, its thickness then being 0·19416 cm.

Field,	24·0	52·2	98·0	114·2	131·0
Effect,	0·0213	0·0288	0·0334	0·0343	0·03133

PLATE II.

Length,	6·26	cm.
Breadth,	2·945	„
Thickness,	0·1497	„

Field,	36·0	61·0	120·0	137·0	146·0
Effect,	0·01746	0·0343	0·0718	0·0806	0·084

The same plate planed to thickness, 0·08277 cm.

Field, . .	15·7	28·2	66·7	94·2	110·0	147·0
Effect, . .	0·0068	0·02114	0·0608	0·0878	0·1045	0·122

PLATE III.

Length, ...	1·99	cm.
Breadth, ...	1·075	"
Thickness, ...	0·12697	"

Field, . .	98·0	138·0	146·0	153·0
Effect, . .	0·0362	0·06246	0·06349	0·06904

The same plate was planed to thickness, 0·09 cm.

Field, ...	33·8	61·0	93·2	110·0	153·0
Effect, ...	0·0125	0·03061	0·04977	0·0609	0·06831

It will be observed that in Plate I. a maximum effect is reached just as was the case with the transverse effect in that plate; and in II. and III. no such maximum effect was reached again agreeing with the transverse current.

The same effect was observed and measured in Plates VIII. and IX.; in those plates which gave a reversal of the transverse current; the result was too small to be measured accurately. In some cases, however, it was noticeable and always directed against the transverse current. In those plates in which the effect was observed, it must be added to the transverse effect to give the latter its proper value.

To find how this effect depended on the current, the field was kept constant, and four different currents used.

Primary Current in Scale Parts.	$\dfrac{\text{Effect in Scale Parts.}}{\text{Primary in Scale Parts.}}$
60·0	0·06458
124·1	0·06305
187·1	0·062
373·0	0·06246

or for the currents used, the effect is proportional to the current.

To explain this effect we must remember that the body carrying the current in a magnetic field is subject to mechanical force and is also heated by the current. According to Joule's law the heating is proportional to the square of the current strength; it cannot, therefore, be due to this heating effect, otherwise its direction would be independent of that of the primary current. We must rather suppose that it is the result of a kind of Peltier effect, arising from the heating of a substance differently deformed in different parts. Should this be so, it might be possible to map out a plate into pressed and stretched regions by observing the direction of the effect in different parts of the plate.

For let us assume, after BIDWELL,* that the plate is deforming in the following manner so that A B represents a compressed part, B F a stretched, and so on ; then if the electrodes are at L and M, the current is in the direction L M ; should they be at N and O the current would be in the direction N O,—that is opposite to the former (fig. F).

If such be the explanation, we should expect to find an electromotive force created in a heated body when it is placed in a magnetic field ; this has been observed by ETTINGSHAUSEN and NERNST.†

* *Phil. Mag.*, 1884.

† WIEDEMANN'S *Annalen*, 1887, Bd. 31.

MR. J. C. BEATTIE ON THE TRANSVERSE EFFECT IN BISMUTH.

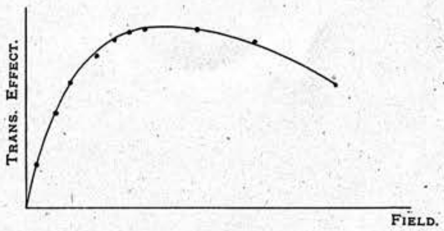
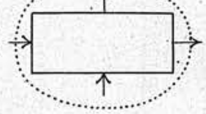


FIG. 1.



(B)

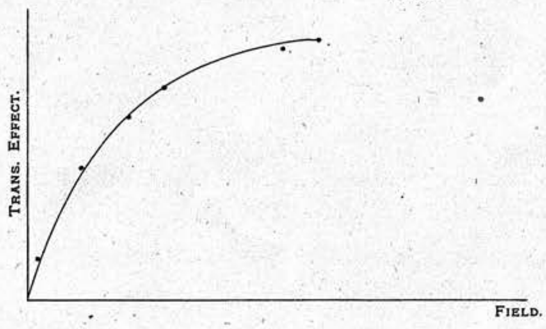
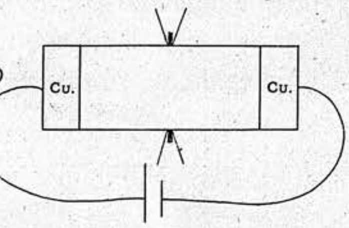


FIG. 2.



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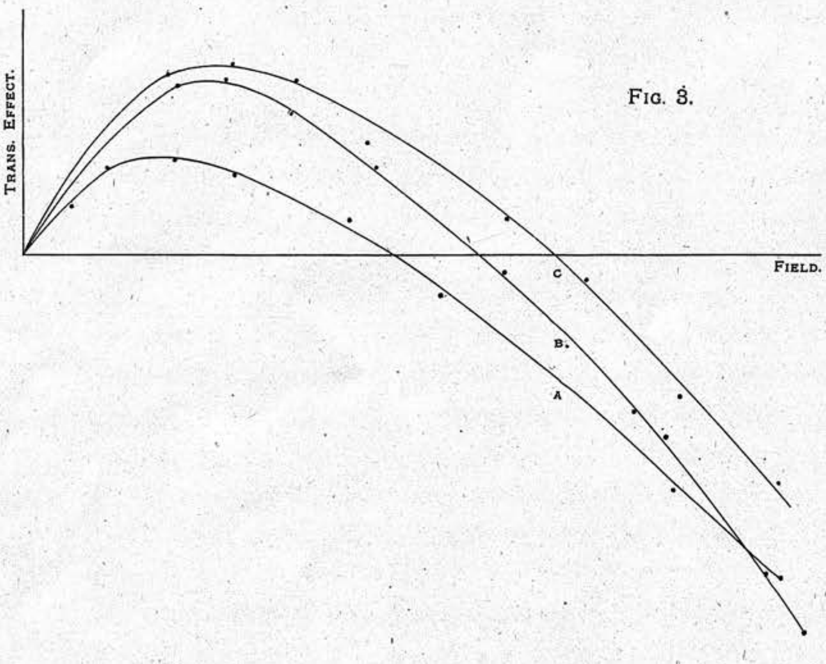
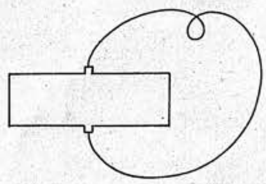
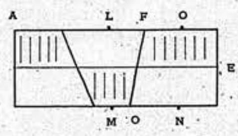


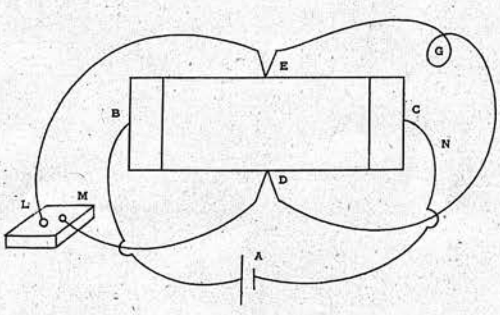
FIG. 3.



(E)



(F)



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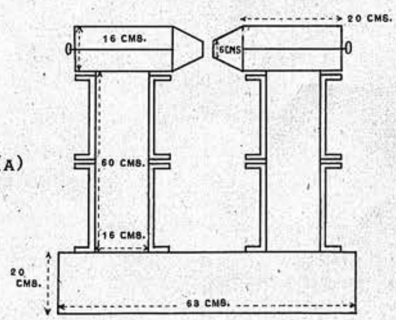

(A)

A. RITCHIE & SON, EDIN.

VII.—*On the Relation between the Variation of Resistance in Bismuth in a Steady Magnetic Field and the Rotatory or Transverse Effect.* By J. C. BEATTIE. (With a Plate.)

(Read 17th June 1895.)

KUNDT* has shown that the transverse effect in iron, cobalt, and nickel is proportional to the magnetisation. Such an effect, where the magnetisation appears in the first power, we shall call a Hall effect. In applying the same method to bismuth, he found that no transverse effect was given by the thin plates of the electrolitically deposited metal used by him. That this absence of transverse effect is not characteristic of all bismuth so prepared has been shown experimentally. The question is, What relation, if any, exists between it and the magnetisation? To settle this it is necessary to compare the transverse effect in any given plate with some other effect in the same plate whose relation to the magnetisation is known. Such an effect is the variation of resistance. GOLDHAMMER† has shown that this latter is proportional to the square of the magnetisation.

The current sent through the plate is called the primary. A thick copper wire was placed in the primary circuit, so that two fixed points in it could be inserted in the galvanometer circuit; the reading thus obtained was used as a measure of the strength of the current. This brings in no error, since the measurements are throughout relative.

By the rotatory or transverse effect is meant the ratio of half the galvanometer deflection (with proper sign), which is obtained when two equipotential or approximately equipotential points on opposite sides of the plate are inserted in the galvanometer circuit, to the strength of the primary current. The numerical value of this effect is denoted by E.

To measure the resistance of the plate, two fixed points or lines in it were inserted in the galvanometer circuit; the reading thus obtained divided by the strength of the primary current was taken to be proportional to the actual resistance of the plate. By this means it is rendered independent of the current strength.

The resistance $n+\Delta n$ of a plate in a steady magnetic field, minus its resistance (n), when no field was there,—that is, Δn can be taken as proportional to the square of the magnetisation.

If the transverse effect is a pure Hall, we shall have

$$c_1\sqrt{\Delta n} = \pm \mathrm{E} \quad . \quad . \quad . \quad (1)$$

Evidently this cannot hold for plates where E attains a maximum value: in such we must use a formula

$$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = \pm \mathrm{E} \quad . \quad (2)$$

In the following experiments a d'Arsonval galvanometer was used. The electromagnet was ring-formed, and was wound with a wire capable of carrying a thirty ampere

* WIEDEMANN'S *Annalen Neue Folge*, Bd. 49, 1893. † *Ibid.*, Bd. 36, 1889.

current; the poles were circular surfaces 60 mm. in diameter and 18 mm. apart. By inserting suitable resistances in the electro-magnet circuit any field required could be obtained.

The field strength was measured by Verdet's method: as the strength does not come directly into the calculations it is given only approximately. The necessary measurements were made some weeks after the other experiments.

The plates used were fixed on to strips of ebonite; at both ends copper of the same breadth and thickness was soldered on, the ends of the copper dipped into pools of mercury. The two pools could be connected with the primary current and with the galvanometer simultaneously; in this way the resistance of the plate perpendicular to the direction of the field was measured. It is to be noted that the resistance of the copper plates comes in, but as this does not vary in a magnetic field, Δn is not affected.

To measure the transverse effect and the resistance along the lines of force two wires arranged as in fig. 1, were soldered on to the two middle points of the sides of the bismuth plate; the ends of these wires dipped into four small mercury pools.

The plates so arranged could be clamped in the field in either of two positions at right angles to one another.

Three different positions of the plate with respect to the lines of force of the field were considered.

Suppose the direction of the field to be parallel to the plane of the paper, and let this be our y-axis: let the z-axis be drawn perpendicularly upwards, the x-axis towards the reader. In the first position (α) the plate's surface was in the xz plane, and the primary current flowed in the z direction. In the second position (β) the plate's surface was in the yz plane, and the primary current flowed in the direction z. The resistance measured in both these cases is the resistance perpendicular to the lines of force of the magnetic field.

In the third position (γ) the plate's surface was in the yz plane, and the primary current flowed in the direction y. With this arrangement the resistance along the lines of force could be measured by sending the primary current in at (1) or (2), while at the same time (3) and (4) were joined to the galvanometer.

It was found, however, that this latter arrangement was not very suitable, and in the greater number of cases another method (fig. δ) was used. The plate was fixed on to another piece of ebonite. Along the sides thick copper wires were soldered throughout the whole length; these served for the primary current. Two other wires were soldered along the length of the plate, but were not in direct contact with the other two: one end of each of these was joined to the galvanometer.

The transverse effect was measured with the plate in position α.

No attempt was made to keep the temperature of the plate constant by using liquids; the temperatures given are the approximate temperatures of the room during the time of the experiment. Between the different experiments, however, a pause was made to allow the plate to cool.

It may be stated that for weak fields the method here given for measuring the resistance is not suitable. For such the Wheatstone bridge or the differential galvanometer would give good results in less time.

Of the different plates used, only one was known to be perfectly pure. With it the following results were obtained :—

Length, . . .	16·75	mm.	Temp. 10° C. $n = \cdot 9521$
Breadth, . . .	7·2		
Thickness, . .	·9		

$$c_1 = \frac{-E}{\sqrt{\Delta n}}$$

Field in cgs. Units.	Transverse Effect.	α Δn	α $\sqrt{\Delta n}$	β Δn	β $\sqrt{\Delta n}$	γ Δn	γ $\sqrt{\Delta n}$	α	β	γ
8,500	−·2311	·2949	·5420	·3144	·5648	·1489	·3858	−·42	−·41	−·60
9,500	−·2455	·3292	·5738	·3556	·5963	·1622	·4027	−·42	−·41	−·60
11,700	−·2627	·4001	·6325	·4333	·6582	·1780	·4219	−·41	−·40	−·62
12,840	−·2708	·4552	·6747	·4717	·6768	·2066	·4545	−·40	−·40	−·59
14,170	−·2974	·4815	·6932	·5258	·7251	·2357	·4855	−·42	−·40	−·61
15,600	−·3071	·5881	·7669	·5684	·7539	·2500	·5000	−·40	−·40	−·61
17,800	−·3271	·6281	·7926	·6252	·7907	·2704	·5389	−·41	−·41	−·60

The relation between the transverse effect and the variation of resistance is a simple one. The constants obtained for the positions α and β are the same : that for γ is different; but it is to be remembered that the results given above depend upon the form of the plate and upon the external resistance inserted in the galvanometer circuit, which differed in position γ from that in α and β. To eliminate the various disturbing effects the quantity $\sqrt{\frac{\Delta n}{n}}$ should be used in equation (1) instead of $\sqrt{\Delta n}$.

The two next plates were made from different supplies of commercial bismuth. In them the numerical value of the transverse effect did not reach a maximum with the fields at disposal.

In Plate II. the simple relation (1) no longer applies; the equation (2) was therefore used.

The average values of the constants c_1 and c_2 were obtained by solving the equations obtained from combining the results for each field with those for every other; the two results immediately before and after were in each case rejected :—that is, with twelve different field strengths, 1 . . . 12, we get eleven equations by combining result 6 say with all others, but of these those derived from 5 and 7 were rejected.

PLATE II.

Length,	16·5	mm.	$n = \cdot 83$
Breadth,	8·0	mm.	Temp. 9° C.
Thickness,	·827	mm.	

$$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = -\mathrm{E}.$$

Field in cgs. Units.	Trans. Effect.	α		β		γ		α		β		γ	
		Δn	$\sqrt{\Delta n}$	Δn	$\sqrt{\Delta n}$	Δn	$\sqrt{\Delta n}$	c_1	c_2	c_1	c_2	c_1	c_2
1,170	− ·1087	·0110	·1049	...		...		...		...		...	
1,840	− ·1270	·0150	·1279	...		...		...		...		...	
2,520	− ·2220	·0460	·2145	...		...		− 1·04	+ ·30	...		...	
3,180	− ·3249	·1041	·3229	·1018	·3190	...		− 1·04	+ ·30	− 1·05	+ ·32	...	
5,030	− ·3840	·1499	·3872	...		·2865	·5352	− 1·04	+ ·30	...		...	
8,500	− ·5147	·2945	·5427	...		·5696	·7540	− 1·04	+ ·31	...		− ·74	+ 1·2
9,500	− ·5264	·3130	·5594	...		...		− 1·04	+ ·30	...		...	
11,740	− ·5679	·3756	·6129	·3805	·6163	·7577	·8704	− 1·04	+ ·30	− 1·04	+ ·30	− ·74	+ 1·1
12,840	− ·6075	·4573	·6761	...		...		− 1·04	+ ·30	...		...	
14,200	− ·6181	·4919	·7013	·4820	·6944	...		− 1·04	+ ·30	− 1·04	+ ·28	...	
15,600	− ·6302	·5300	·7215	...		...		− 1·04	+ ·30	...		...	
17,780	− ·6600	·6081	·7798	·6023	·7716	1·0062	1·0031	− 1·04	+ ·30	...		− ·75	+ 1·2
							Average,	− 1·04	+ ·31	− 1·04	+ ·31	− ·75	+ 1·2

In Plate IX. the relation between the transverse effect and the resistance variation perpendicular to the field was alone considered. Owing to the size of the plate the arrangement —ε— was somewhat different. The plate was fixed on to stiff cardboard, the transverse electrodes were soldered to the middle points of the sides, the primary electrodes to the middle points of the ends. The plate was then placed with its surface perpendicular to the lines of magnetic induction and kept clamped in this position.

We see that in these two plates we have not to deal with a pure Hall effect only; we have in addition a second effect, which is positive and proportional to $(\Delta n)^{\frac{3}{2}}$ that is to the magnetisation cubed.

The results for Plate IX. and for all the plates showing two effects can be represented graphically; the resistance variation is measured along the horizontal axis. Along the perpendicular axis the values of $\dfrac{-\mathrm{E}}{\sqrt{\Delta n}}$ are laid down; the connecting curve is a straight line, which, when produced to meet the perpendicular axis, gives the value of c_1. To obtain such a curve for any plate, at least two direct measurements of the

PLATE IX.

Length, . . . 60·1 mm.
Breadth, . . . 30·3 mm.
Thickness, . . . 3·1785 mm.

$n = 1·4052$
Temp. 14° C.

Field.	Trans. Effect.	δ Δn	δ $\sqrt{\Delta n}$		$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = E.$ c_1	c_2
1,340	− ·0469	·0240	·1549		...	...
3,350	− ·0943	·1020	·3194		− ·30	+ ·097
5,030	− ·1202	·1760	·4195		− ·30	+ ·107
6,700	− ·1505	·2975	·5454		− ·30	+ ·098
8,800	− ·1700	·4190	·6473		− ·30	+ ·101
11,300	− ·1879	·5823	·7631		− ·30	+ ·101
14,750	− ·1999	·7735	·8795		− ·30	+ ·100
17,780	− ·2072	·9870	·9985		− ·30	+ ·098
				Average	− ·30	+ ·100

transverse effect and of the variation of resistance must be made; with these we obtain two points on our line. For any other variation of resistance value we can then find the transverse effect and resolve it into a pure Hall effect and this second effect.

Evidently the graphic method could also be applied to determine c_1 and c_2 in the first instance, instead of the method of equations used in this paper.

The next plates used were two in which the transverse effect attains a maximum numerical value.

In these two plates also, which were made from the same two supplies of bismuth, we have two effects.

Finally, two plates were considered in which the second effect is so great as to completely mask the true Hall, so that even with the low fields at disposal the transverse effect changes sign.

The results of experiments with Plates IB. and VI. by a different method:—viz., that described in paper, "On Hall Effect and some Related Effects in Bismuth," were qualitatively the same. For IB. the values of c_1 and c_2 were respectively $c_1 = -·45$ $c_2 = +4·4$ and for VI. $c_1 = -·238$ $c_2 = +3·3$.

We may sum up the results so far obtained by saying that in the pure bismuth plate the Hall effect alone is present: it is proportional to $(\Delta n)^{\frac{1}{2}}$ and is negative in sign. In the other plates the transverse effect is composed of two effects, the pure Hall and another, positive in sign and proportional to $(\Delta n)^{\frac{3}{2}}$. In different plates this effect appears in different magnitude; in some it is relatively so small, compared to the Hall effect, that it does not, with the fields at disposal, cause the total effect to decrease numerically. In others, again, it produces this; and in a third class it

PLATE VIII.

Length, ... 24·0 mm.
Breadth, ... 12·0 mm.
Thickness, ... 1·1365 mm.

$n = 1{\cdot}3967$
Temp. 15° C.

$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = -\text{E}.$

Field.	Trans. Effect.	α		β		γ		δ	
		Δn	$\sqrt{\Delta n}$	Δn	$\sqrt{\Delta n}$	c_1	c_2	c_1	c_2
1,340	− ·10221	·0320	·1780	...	...	...	...	...	...
3,350	− ·19552	·1253	·3539	·0452	·2126	...	...	...	...
5,030	·23712	·2029	·4500	·0712	·2668	− ·58	+ ·27	− ·96	1·2
6,700	− ·28526	·3244	·5697	·1135	·3369	− ·58	+ ·25	− ·96	1·1
11,300	− ·33459	·6067	·7789	·1943	·4409	− ·58	+ ·25	− ·96	1·1
12,840	− ·33879	·7199	·8489	...	...	− ·58	+ ·25	...	..
14,750	− ·33456	·7803	·8833	·2416	·4914	− ·58	+ ·26	− ·96	1·2
17,780	− ·3147	·9709	·9853	·2699	·5195	− ·57	+ ·26	− ·98	1·4
					Average	− ·58	+ ·26	− ·97	1·2

PLATE IA.—ARRANGEMENT ε.

Length, ... 50 mm.
Breadth, ... 28·25 mm.
Thickness, ... 1·305 mm.

$n = 1{\cdot}056$
Temp. 15° C.

$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = -\text{E}.$

Field.	Trans. Effect.	Δn	$\sqrt{\Delta n}$	c_1	c_2
8,500	− ·2902	·2529	·5029	...	...
11,740	− ·2900	·3262	·5712	− ·83	+ ·98
12,840	− ·2898	·3274	·5722	− ·81	+ ·94
14,170	− ·2739	·4121	·6419	− ·83	+ ·95
15,600	− ·2621	·4321	·6573	− ·80	+ ·99
17,780	− ·2503	·4816	·6939	− ·82	+ ·95
			Average	− ·82	+ ·96

PLATE XII.

Length, . . . 24·0 mm.
Breadth, . . . 11·75 mm.
Thickness, . . . ·95 mm.

$n = 2{\cdot}08$
Temp. 14° C.

$c_1(\Delta n)^{\frac{1}{2}} + c_2(\Delta n)^{\frac{3}{2}} = \pm E.$

Field.	Trans. Effect.	α Δn	α $\sqrt{\Delta n}$	β Δn	β $\sqrt{\Delta n}$	α c_1	α c_2	β c_1	β c_2
1,340	−·0475	·0139	·1179	...		...		...	
2,680	−·07266	·0373	·1931	...		...		...	
3,350	−·08131	·0517	·2274	·0382	·1955	−·41	+1·2	...	
5,030	−·0976	·0776	·2786	·0559	·2383	−·41	1·2	−·50	+2·2
6,700	−·08922	·1393	·3732	·0900	·3000	−·42	1·3	−·50	+2·3
8,820	−·07009	·1906	·4366	·1282	·3580	−·41	1·3	−·50	2·4
14,750	−·00519	·3318	·5760	...		−·43	1·2	...	
17,780	+·04702	·3816	·6177	·2322	·4819	−·42	1·3	−·50	2·4
					Average,	−·42	+1·3	−·50	+2·5

PLATE X.

Length, . . . 38·8 mm.
Breadth, . . . 20·5 mm.
Thickness, . . . 1·112 mm.

1·8041
Temp. 9° C.

$c_1\sqrt{\Delta n} + c_2(\Delta n)^{\frac{3}{2}} = \pm E.$

Field.	Trans. Effect.	ϵ Δn	ϵ $\sqrt{\Delta n}$	c_1	c_2
1,840	·01708	·0148	·1217	...	
2,520	−·01944	·0221	·1487	...	
3,180	−·01974	·0400	·2000	−·16	+1·7
4,100	−·01784	·0455	·2133	−·16	1·8
8,500	+·01361	·1300	·3606	−·17	1·6
10,800	+·02650	·1502	·5875	−·15	1·6
12,840	+·06014	·1830	·4278	−·15	1·7
14,170	+·07676	·1977	·4448	−·16	1·7
15,600	+·09602	·2033	·4575	−·16	1·8
17,780	+·12024	·2215	·4707	−·16	1·8

is present to such an extent that in the end it gives its sign to the total transverse effect.

This second effect is not to be confounded with the thermo-magnetic effect observed by ETTINGSHAUSEN and NERNST: the latter is evidently proportional to $(\Delta n)^{\frac{1}{2}}$ and positive in sign.

The anomalous behaviour of the transverse effect in bismuth—which is hidden, if the effect be represented in terms of the rotatory co-efficient R—has also been observed by ETTINGSHAUSEN and NERNST.* They found that in a specimen of pure bismuth, E obtained a maximum value,—that is, both effects may appear in pure bismuth. Again, ETTINGSHAUSEN † has shown that, in an alloy of tin and bismuth, the transverse effect changes sign. At high fields, when little tin is present: at lower, when more tin is added until when the alloy contains 6 % tin, 94 % bismuth, the positive sign alone is present. The explanation lies in the presence of this second effect, which increases relatively to the pure Hall effect as the proportion of tin to bismuth increases.

It is interesting to note that the relation between transverse effect and resistance variation holds, no matter what the percentage increase of resistance is, or how much the transverse effects expressed in terms of R vary in the different plates.

So far, the transverse effect has only been observed when the electrodes are at the middle points of the sides. A number of experiments were next made with these electrodes at different parts, while still kept opposite each other. In Plate IA. the numerical value of the transverse effect was found to have a maximum value with the electrodes in the middle; for other positions it was less. The greater the distance from the middle points, the greater was the decrease. Next, the same plate was cut along the middle line for about half its length. The electrodes were fixed at $a\ b$, $c\ d$, $e\ f$, respectively, fig. 2, and the effect was found to be greatest at $a\ b$, less at $e\ f$ and $c\ d$. Finally, another slit was made along the middle line, and the electrodes were placed at f and g, fig. 3. The effect was qualitatively the same, but quantitatively less.

The question to settle now is, Whether this decrease is due to a decrease in the pure Hall effect? in the second effect? or in both? If we take the ratio $\frac{\Delta n}{n}$ for any one plate, we get a number which may be looked on as characteristic of that plate; it is independent of its dimensions, and depends only on its properties. If these are the same throughout—which we assume to be the case—and if we neglect the slight disturbances due to the fact that the temperature is not absolutely constant throughout the experiments, this number may be used to divide the transverse effect into its two constituents, and to give the relative values of these for any one plate, no matter how it is modified in size or shape. That is, we now apply the equation

$$c_1\left(\frac{\Delta n}{n}\right)^{\frac{1}{2}} + c_2\left(\frac{\Delta n}{n}\right)^{\frac{3}{2}} = \pm \mathrm{E} \qquad . \qquad . \qquad (3).$$

* *Sitz. bericht der kais. Akad. der Wissenschaft*, Wien, 1886.

† *Sitz. bericht der kais. Akad.*, Wien, 1887.

When this is done, we find that the decrease in the transverse effect, when the plate has the form fig. 3, is due to a decrease in both effects. Similarly, the decrease, as we move towards the ends, is also due to a decrease in both.

Another plate, VIIIA, was used next. The transverse electrodes were first placed in the middle, then at points 2 mm. from the ends, the transverse effect in the second position was the smaller. The results, treated as in IA, showed that the decrease was again due to a decrease in both effects.

PLATE VIIIA.

Length, . . .	44·0	mm.	Temp. 15° C.
Breadth, . . .	21·0	mm.	
Thickness, . . .	1·05	mm.	

$$c_1\left(\frac{\Delta n}{n}\right)^{\frac{1}{2}} + c_2\left(\frac{\Delta n}{n}\right)^{\frac{3}{2}} = -\mathrm{E}.$$

Field.	Trans. Effect with Elect. in Middle.	Trans. Effect with Elect. near Ends.	$\frac{\Delta n}{n}$	$\frac{\sqrt{\Delta n}}{n}$	Electrodes in Middle. c_1	Electrodes in Middle. c_2	Electrodes near Ends. c_1	Electrodes near Ends. c_2
3,350	−·1616	−·1463	·0616	·2482	−·69	+·74	−·63	+·68
6,700	−·2371	−·2107	·1716	·4142	−·67	+·72	−·62	+·64
11,300	−·2612	−·2348	·3091	·5559	−·68	+·70	−·63	+·68
17,780	−·2377	−·2066	·5009	·7077	...	...	...	...

In Plate X. the electrodes were first soldered on to middle points of the sides, then to points 4 mm. from the end; the variation, however, was so small that no conclusion could be drawn as to its cause. This plate was also slit along the middle line, so that it had the form given in fig. 3, the results were qualitatively the same, but showed a decrease in E. This same plate, after being used for some time, showed a change in the field strength necessary to make the effect vanish. A higher field became necessary. The change was very small, and the application of equation 3 showed that the pure Hall effect had increased, while the second effect remained practically constant.

In a former paper it was shown that, in plates for which the transverse effect changes sign, the field strength at which the effect vanishes is raised when the plate is hammered or filed. To find what change in the pure Hall or in the second effect is concerned in this, the results obtained with Plate IB were examined; and it would appear that the pure Hall effect varies, while the second remains practically constant.

PLATE IB.

Length,	44·7	mm.
Breadth,	20·8	mm.
Thickness,	1·2235	mm.

$$c_1\sqrt{\frac{\Delta n}{n}}+c_2\left(\sqrt{\frac{\Delta n}{n}}\right)^3=\pm\text{E}.$$

Field (about).	Trans. Effect.	$\frac{\Delta n}{n}$	$\sqrt{\frac{\Delta n}{n}}$	c_1	c_2
3,864	− ·07693	·0408	·2019	...	...
8,694	− ·02957	·1002	·3165	− ·58	+ 4·8
16,284	+ ·15039	·1828	·4276	− ·57	+ 5·1
19,320	+ ·21259	·2203	·4639	− ·57	+ 4·7

The plate was next hammered and, approximately, the same field strengths used. The results were now—

$$c_1\left(\frac{\Delta n}{n}\right)^{\frac{1}{2}}+c_2\left(\frac{\Delta n}{n}\right)^{\frac{3}{2}}=\pm\text{E}.$$

Field.	Trans. Effect.	$\Delta n/n$ $\sqrt{\Delta n}/n$	c_1	c_2
As in last.	− ·0126	As in last.	...	...
Do.	− ·05166	Do.	− ·73	+ 5·7
Do.	+ ·11249	Do.	− ·72	+ 5·3
Do.	+ ·19027	Do.	− ·73	+ 5·2

Finally, the plate was filed down, its thickness being reduced to ·665 mm. which, when the variation of resistance is taken into account, gives us

$$c_1=-\cdot 76\ c_2=+5\cdot 4.$$

Field.	Trans. Effect.	c_1	c_2
3,864	− ·21699	...	...
8,694	− ·13521	− 1·52	10·9
16,284	+ ·17013	− 1·50	10·4

The same results were obtained with Plate IX. It was first used when its thickness was 3·18 mm. The constants for this were—

$$c_1=-\cdot 18\ c_2=+\cdot 085.$$

Next it was filed down till a thickness 1·56 mm. was obtained. The constants were now—

$$c_1 = -\cdot 42 \quad c_2 = +\cdot 171.$$

That is, the pure Hall effect has slightly increased ; the second effect has remained constant.

From the above we conclude that when a plate is slit along the middle line, as IA, the transverse effect changes in numerical value, but not in sign ; the fact also that the effect decreases, but qualitatively does not change as we pass from the middle towards the ends, admits of a similar explanation. For, suppose we have a plate with electrodes (transverse) at *a* and *b* (fig. 4), the rotatory effect may be represented as in the figure. When the slits are made, several lines are interrupted (fig. 5) ; and when we approach the ends, the complete number of lines is given only on one side of the connecting line. The single safe conclusion to be drawn seems to be that the state of the plate, when it gives a transverse effect, is symmetric with respect to the middle line of its length.

The causes of the pure Hall effect and of the second effect seem to be very intimately connected. Only by hammering or by filing a plate does it seem possible to vary one without varying the other.

Evidently the relations obtained between the transverse effect and the resistance variation for the various plates do not allow us to compare the values of the constants in different plates, even when we use $\frac{\Delta n}{n}$; for this latter is a variable standard, depending on the plate and the temperature.

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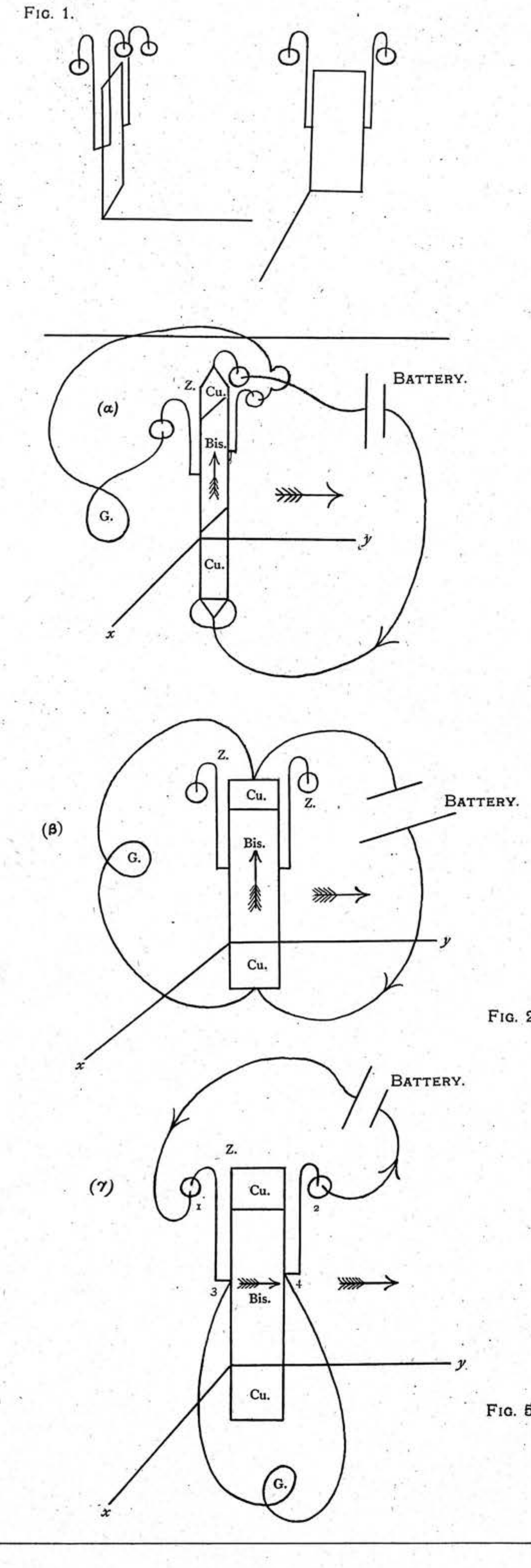
FIG. 1.



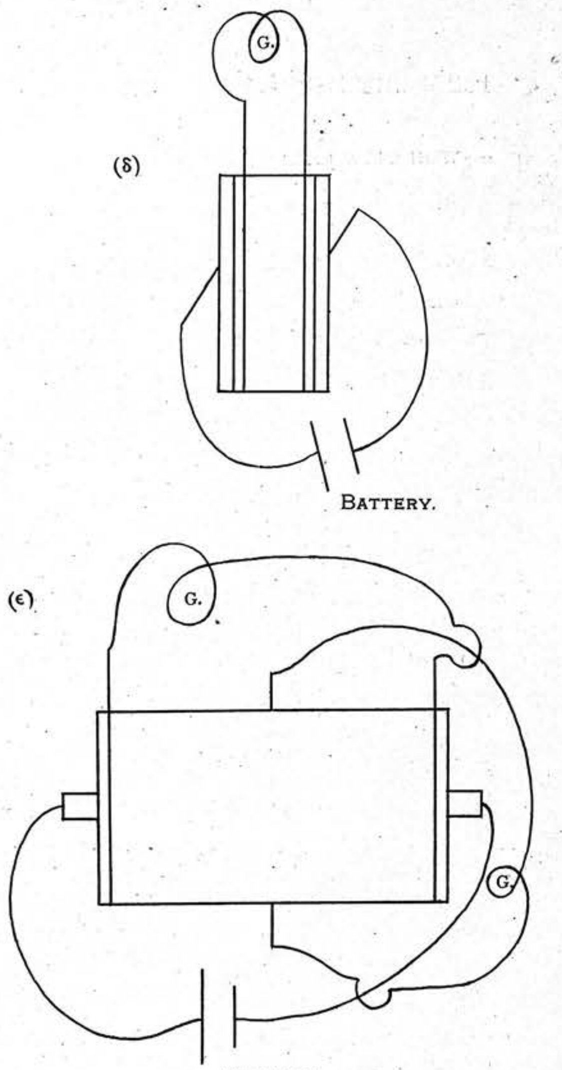


FIG. 2.

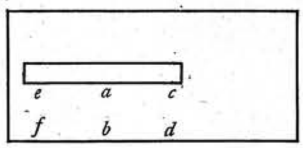


FIG. 3.

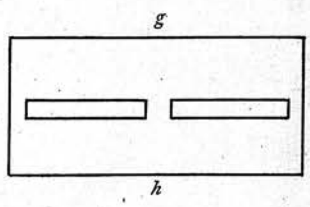


FIG. 4.

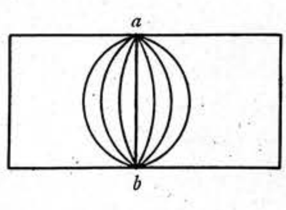


FIG. 5.

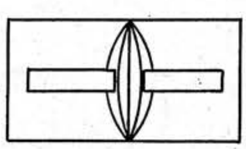


FIG. 6.